Stochastic Chambolle Pock

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Problem

F, G convex functions, A matrix, b vector.

Problem :

$$\min_x F(x) + G(x) \quad \text{such that} \quad Ax = b \tag{1}$$

Chambolle-Pock Algorithm is a splitting algorithm that solves (1) via a minimax formulation

$$\begin{aligned} x_{n+1} &= \mathrm{prox}_{\gamma G}\left(x_n - \gamma(\nabla F(x_n) + A^T y_n)\right) \\ y_{n+1} &= y_n + \gamma\left(A(2x_{n+1} - x_n) - b\right) \end{aligned}$$

rewrite it $z_n = (x_n, y_n)$ and

$$z_{n+1} = T_{F,G,A,b}(z_n)$$

Stochastic Setting

Problem :

$$\min_x F(x) + G(x) \quad \text{such that} \quad Ax = b \tag{2}$$

with

$$F(x) = \mathbb{E}_\xi(f(x,\xi)), G(x) = \mathbb{E}_\xi(g(x,\xi)), A = \mathbb{E}_\xi(A(\xi)), b = \mathbb{E}_\xi(b(\xi))$$

where ξ random variable.

It means that $f(\cdot,\xi), g(\cdot,\xi)$ are random functions, $A(\xi)$ random matrix, $b(\xi)$ random vector.

Stochastic Chambolle-Pock

Assume that we know how to compute $f(\cdot, \xi), g(\cdot, \xi), A(\xi), b(\xi)$ for every ξ but we don't know how to compute F, G, A, b.

Stochastic Chambolle Pock :
Let (ξ_n) i.i.d copies of ξ.

$$z_{n+1} = T_{f(\cdot,\xi_{n+1}),g(\cdot,\xi_{n+1}),A(\xi_{n+1}),b(\xi_{n+1})}(z_n)$$

- Convergence rate?
- Applications?